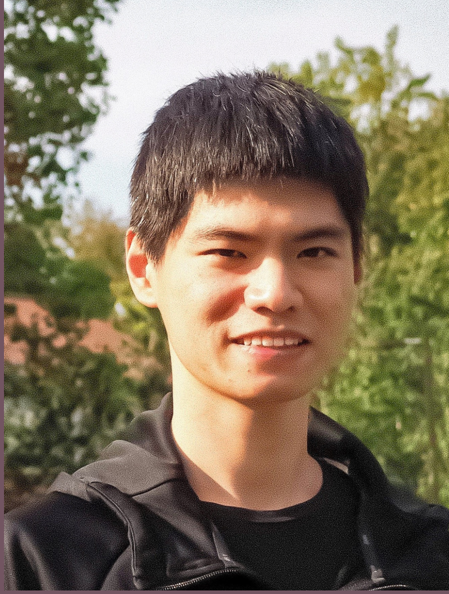
The Hyperbolic Positive Mass Theorem and Volume Comparison Involving Scalar Curvature



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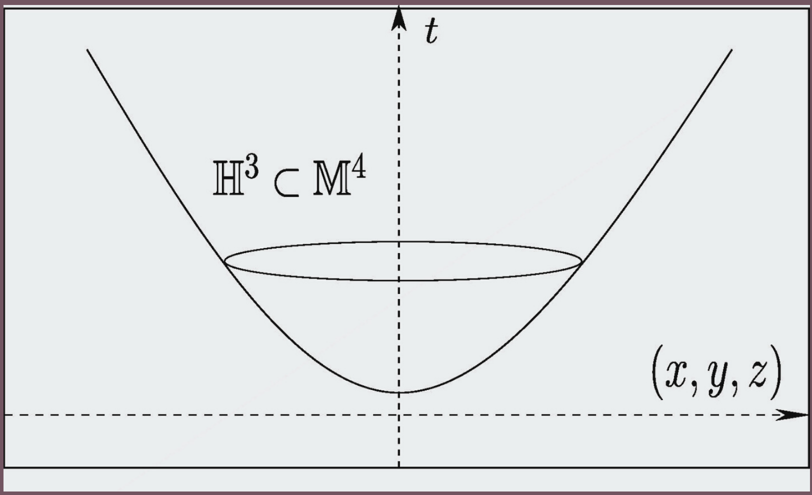
PhD Thesis

According to general relativity, gravity is not a force, but a consequence of the curved spacetime. The fundamental idea in general relativity is that matter curves spacetime. The reason why the earth circles around the sun is attractive gravity which is due to positive masses. This statement is known as the positive mass theorem. It asserts that for an isolated system, nonnegative local energy density implies nonnegative total mass. It is a collection of foundational results in general relativity and differential geometry. The first proof was obtained by Schoen and Yau. Another approach was given by Witten. However, some formulations of the positive mass theorem are still open.



Using a novel technique developed by Stern, we prove the hyperbolic positive mass theorem in the spacetime setting, as well as some rigidity cases. A new interpretation of mass is introduced in this context. We also obtain a lower bound for the total mass without assuming the nonnegativity of the local energy density. Another part of my dissertation studies the relation between scalar curvature and volume. I prove some volume comparison theorems involving scalar curvature for dimensions greater than three.