A summary of the basic integral theorems of vector calculus.

Part One. \( n = 2. \)

Suppose
\[
\mathbf{F} = P\mathbf{i} + Q\mathbf{j}
\]

**Green’s Theorem.** Suppose \( R \) is a bounded region in \( \mathbb{R}^2 \) with boundary \( C \). Suppose \( \mathbf{T} \) is a unit tangent vector to \( C \) which points in the counterclockwise direction on the outer part of \( C \) and in the clockwise direction on the inner part of \( C \). Then
\[
\int_C \mathbf{F} \cdot \mathbf{T} \; ds = \iint_R \nabla \times \mathbf{F} \; dA.
\]

\( \nabla \times \mathbf{F} \) here is, by definition, the scalar \( Q_x - P_y. \)

**Remark.** This may also be written
\[
\int_C P \, dx + Q \, dy = \iint_R Q_x - P_y \, dx \, dy
\]
where \( C \) is oriented as before.

**The Divergence Theorem.** Suppose \( R \) is a bounded region in \( \mathbb{R}^2 \) with boundary \( C \). Suppose \( \mathbf{n} \) is the outward pointing unit exterior normal to \( R \) along its boundary \( C \). Then
\[
\int_C \mathbf{F} \cdot \mathbf{n} \, ds = \iint_R \nabla \cdot \mathbf{F} \; dA.
\]

Part Two. \( n = 3. \) Suppose
\[
\mathbf{F} = P\mathbf{i} + Q\mathbf{j} + R\mathbf{k}.
\]

**Stokes’ Theorem.** Suppose \( S \) is a surface in \( \mathbb{R}^3 \) with boundary \( C \) and unit normal \( \mathbf{n} \). Suppose \( \mathbf{T} \) is the unit tangent field along \( C \) such that \( \mathbf{n} \times \mathbf{T} \) points into \( S \). Then
\[
\int_C \mathbf{F} \cdot \mathbf{T} \; ds = \iint_S (\nabla \times \mathbf{F}) \cdot \mathbf{n} \; dA.
\]

**Remark.** This may also be written
\[
\int_C P \, dx + Q \, dy + R \, dz = \iint_S (R_y - Q_z) \, dy \, dz + (P_z - R_x) \, dz \, dx + (Q_x - P_y) \, dx \, dy
\]
where \( S \) is oriented as before.

**The Divergence Theorem.** Suppose \( T \) is a bounded region in \( \mathbb{R}^3 \) with boundary \( S \). Suppose \( \mathbf{n} \) is the outward pointing unit exterior normal to \( T \) along its boundary \( S \). Then
\[
\iiint_T \mathbf{F} \cdot \mathbf{n} \; dA = \iiint_T \nabla \cdot \mathbf{F} \; dV.
\]