The following exercises represent some of the most important concepts from Math 105L. Students who are beginning Math 106L should be familiar with these concepts, and the problems that are presented here are meant to serve as a review of those concepts. Work together with your partner to complete the statements or answer the questions. Use a separate piece of paper if you need more room. If you cannot complete this work in class today, arrange to meet your partner before class Monday. Hand in one neatly written copy of this on Gradescope by 8am Monday.

The 105L worksheets on which you can find information on each question is given by each question. You may find your 105L worksheets on Gradescope.

1. (105L worksheet: The Derivative Function) The derivative of a function $f(x)$ is defined to be:
$$f'(x) = \lim_{h \to 0}$$

2. (105L worksheet: Linear Approximations) Find the linear approximation to the function $f(x) = xe^{kx}$ at $x = 0$. Assuming that $k > 0$, does the linear approximation overestimate or underestimate xe^{kx} near 0? Explain your answer carefully.

3. (105L worksheets: The Derivative Function, The Second Derivative, and Using First and Second Derivatives)

   On the axes to the right, identify which graph represents $f(x)$, which represents $f'(x)$ and which represents $f''(x)$. Briefly explain your reasoning.
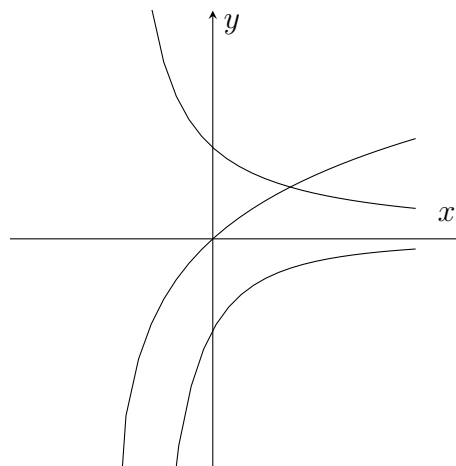
4. (105L Linear modeling lab, Chain Rule Worksheet) Using the table below, estimate $\frac{d}{dx}f(g(x))$ at $x = 1.3$ as closely as possible. Explain your reasoning. (Hint: You will need the chain rule, but you will also need to estimate derivatives. What is the ideal way to estimate them from data?)

x	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0
$f(x)$	1.4	1.2	1.1	1.0	1.3	1.4	1.7	1.5	1.6	2.1	2.2
$g(x)$	0.9	1.4	2.2	2.0	1.9	1.5	2.2	3.1	2.5	2.2	2.3

5. (105L log plots lab) Suppose you have collected some data for two variables x and y.

   (a) Fill in the blanks: If the plot of _____ vs. _____ (the _____-log plot) is linear, then an exponential function fits the data well.

   (b) Fill in the blanks: If the plot of _____ vs. _____ (the _____-log plot) is linear, then a power function fits the data well.

6. (105L worksheet: Related Rates) Suppose that you are watching a rocket take off. You are $1,500$ meters from the launch site. At time t seconds after launch, the height of the rocket in meters is given by $h(t) = 1.39t^2 - 1.65t$. Find the rate at which the distance between you and the rocket is changing at time 10 seconds after launch. You may assume that the rocket rises vertically, and that the ground is flat.

7. (105L worksheets: Power Functions and Polynomials, Rational Functions) A general rational function can be written as

$$f(x) = \frac{a_n x^n + a_{n-1} x^{n-1} + \ldots + a_2 x^2 + a_1 x + a_0}{b_m x^m + b_{m-1} x^{m-1} + \ldots + b_2 x^2 + b_1 x + b_0}.$$

Fill in the following blanks:

- If $m > n$, then $f(x)$ has a horizontal asymptote at _____.

- If m___n, then $f(x)$ has a horizontal asymptote at _____.
  (The blanks here should have different answers than in the first bullet point).

8. (105L worksheet: Differentiability) Fill in the blanks with the words *continuous* and *differentiable*:

- If $f(x)$ is not _____ at $x = a$, then it is not _____ at $x = a$;

- If $f(x)$ is _____ at $x = a$, then it is _____ at $x = a$;

- It is possible for $f(x)$ to be _____ at $x = a$, but not _____ there.

9. (105L worksheets: Exponential Functions, Logarithms) Fill in the blanks:

$$f(x) = 10^x \qquad f(x) = \log x$$

	$f(x) = 10^x$	$f(x) = \log x$
Domain	_____	_____
Range	_____	_____
x intercept	_____	_____
y intercept	_____	_____
Horiz. asymptote	_____	_____
Vert. asymptote	_____	_____

10. (105L worksheets Logarithms, Differentiating Logarithmic Functions)

   (a) $\ln x$ is the inverse of what function?

   (b) Using the derivative of its inverse, show why $\frac{d}{dx} \ln x = \frac{1}{x}$

11. (105L Derivatives and Roots lab) In each of the following, draw a graph of a *differentiable* function $f(x)$ with domain $(-\infty, \infty)$ that satisfies the given conditions. If it is not possible, explain why. Your graph should make it clear what the behavior of $f(x)$ is as x approaches $-\infty$ and ∞.

   (a) $f'(x)$ has three distinct zeroes and $f(x)$ has one zero.

   (b) $f(x)$ has three distinct zeroes and $f'(x)$ has one zero.

12. (105L worksheets Using First and Second Derivatives, Optimization - Global Extrema) Without using a calculator, find the the global max and min of $f(x) = x^2 e^{2x}$ on $[0, \infty)$.