

## Review

When resources are limited, we expect that that relative growth rate gets smaller as population grows. In general, when there is a limit  $L$  to how large  $P(t)$  can get, a good model is the *logistic model*:

$$\text{relative growth rate} = \frac{1}{P} \frac{dP}{dt} = k \left( 1 - \frac{P}{L} \right).$$

The solution to the logistic differential equation with initial condition  $P(0) = P_0$  is the logistic function:

$$P(t) = \frac{L}{1 + Ae^{-kt}} \quad \text{with} \quad A = \frac{L - P_0}{P_0}.$$

Also for a logistic function, the maximum value of  $\frac{dP}{dt}$  occurs when  $P = \frac{L}{2}$ , and the time to the max rate of change is  $\frac{1}{k} \ln A = \frac{1}{k} \ln \frac{L - P_0}{P_0}$ .

## Running

Jane is training for a race. She currently runs at a 3 mile per hour pace. Jane begins training on Jan. 1, 2022; her pace improves dramatically, although she will never be able to run faster than 7 miles per hour. Let  $v(t)$  be Jane's pace (in miles per hour)  $t$  days after Jan. 1.

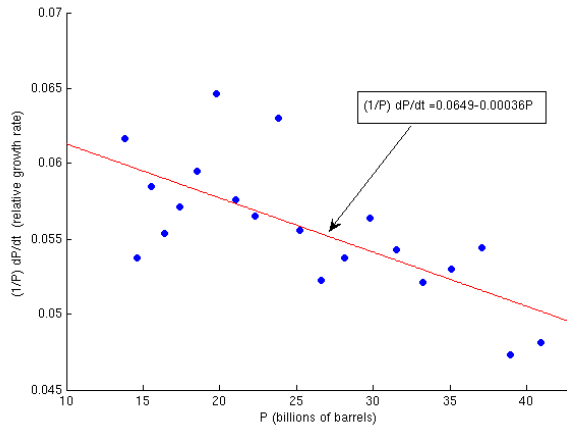
1. Assuming that her pace  $v(t)$  grows according to a logistic model, write down an IVP for  $v(t)$ . Your DE will include the constant  $k$ , as you do not yet know its value.
  
  
  
  
  
  
  
  
  
  
2. After 100 days of training, Jane runs a 5 mile/hr pace. Find an explicit formula for  $v(t)$ , then plot it. How long will it take for Jane to have achieve a 6 mile/hr pace? (Hint: you will have to calculate  $k$ ...)

## US Oil Production

Let  $P(t)$  be the **total** amount of oil (in billions of barrels) produced in the US since 1859, the year that oil production began. Assume that  $P(t)$  grows according to the logistic model  $\frac{dP}{dt} = kP \left(1 - \frac{P}{L}\right)$ .

3. What is the meaning of  $L$  in this context?

4. The data below are based on estimates for both  $P$  and  $\frac{dP}{dt}$  for the years 1931-1950. A best fit line is shown with its equation. Use it to estimate  $k$  and  $L$ . (Hint:  $\frac{1}{P} \frac{dP}{dt} = k \left(1 - \frac{P}{L}\right)$ . The expression on the right of that equation is the best fit line...)



5. If we let 1950 be time  $t = 0$ , then  $P(0) = 40.9$  billion barrels. Find a formula for  $P(t)$ . You may use the solution to the logistic equation given at the top of the previous page.

6. According to this model, what year will oil production peak in the US and what will be the peak oil production amount? (Hint: if total oil produced is  $P(t)$ , what is oil production per year? Now use the info at the top of the previous page.)

(Note: US oil production actually peaked at 3.5 billion barrels per year in 1970)