Make sure you are computing in *radians*. We will rarely use degrees from now on.

Graphs of $\sin(x)$ and $\cos(x)$

1. Since $\sin(x)$ and $\cos(x)$ are defined by using our unit circle, they repeat themselves every _____. In other words:

   - $\sin(x) = \sin(x + \underline{\ \ }) = \sin(x + \underline{\ \ }) = \ldots$ and
   - $\cos(x) = \cos(x + \underline{\ \ }) = \cos(x + \underline{\ \ }) = \ldots$

2. Use Wolfram Alpha to draw the graphs of $\sin(x)$ and $\cos(x)$ below (on the same set of axes), with domain $[-2\pi, 2\pi]$ (You can literally type '`sin(x) x=-2pi to 2pi`' into Wolfram Alpha):

The Tangent Function

3. We defined $\tan\theta = \dfrac{\text{opp}}{\text{adj}} = \dfrac{\sin\theta}{\cos\theta}$. Using Wolfram Alpha, draw the graph of $\tan x$ with domain $[-2\pi, 2\pi]$ below:

4. (a) What is the period of $\tan x$?

   (b) For what values of x is $\tan x$ positive? Negative? Zero?

   (c) Where does $\tan x$ have vertical asymptotes?

   (d) Does $\lim\limits_{x \to \frac{\pi}{2}} \tan x$ exist? Why or why not?

Trig Identities

5. (a) On the unit circle below, label the coordinate of the point marked on the circle in terms of A:
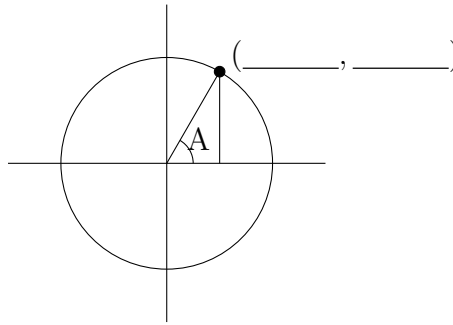
(b) Use Pythagoras to derive an identity relating the lengths of the sides of the right-angled triangle in the diagram to the length of its hypotenuse:

The identity you just wrote down above is the most basic trigonometric identity, from which many others can be derived. Before we do so, we'll introduce three additional trig functions:

- $\sec\theta = \frac{1}{\cos\theta} = \frac{\text{hyp}}{\text{adj}}$ - The *secant* function;

- $\csc\theta = \frac{1}{\sin\theta} = \frac{\text{hyp}}{\text{opp}}$ - The *cosecant* function;

- $\cot\theta = \frac{1}{\tan\theta} = \frac{\text{adj}}{\text{opp}} = \frac{\cos\theta}{\sin\theta}$ - The *cotangent* function.

6. Using the identity you derived above, show that the following identities are true:

(a) $1 + \cot^2 x = \csc^2 x$

(b) $\tan^2 x + 1 = \sec^2 x$

Angle Addition Formulas

We will not prove the following, though they are true and will be very useful:

- $\sin(A + B) = \sin A \cos B + \cos A \sin B$

- $\sin(A - B) = \sin A \cos B - \cos A \sin B$

- $\cos(A + B) = \cos A \cos B - \sin A \sin B$

- $\cos(A - B) = \cos A \cos B + \sin A \sin B$

7. (a) Use the formulas above to show that $\sin(2A) = 2\sin A \cos A$. (Hint: $2A = A + A$)

   (b) Show that $\cos(\frac{\pi}{2} - x) = \sin(x)$.

Two Important Limits

8. (a) Using Wolfram Alpha, draw the graph of $f(x) = \frac{\sin x}{x}$ below:

   (b) Is $f(x)$ defined at $x = 0$? Why or why not?

   (c) By plugging in numbers near 0, complete the following:

   $$\lim_{x \to 0} \frac{\sin x}{x} = \underline{\hspace{1cm}}.$$

9. (a) Using Wolfram Alpha, draw the graph of $g(x) = \frac{\cos x - 1}{x}$ below:

   (b) Is $g(x)$ defined at $x = 0$? Why or why not?

   (c) By plugging in numbers near 0, complete the following:

   $$\lim_{x \to 0} \frac{\cos x - 1}{x} = \underline{\hspace{1cm}}.$$

We will not prove that the limits you wrote down above are true, but we will use them when doing calculus with trig functions.

Homework Questions

1. Use the formula for $\cos(A+B)$ to show that $\cos(2x) = \cos^2(x) - \sin^2(x)$. Then show that $\cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x$. (Hint: use $\sin^2 x + \cos^2 x = 1$ for the second part.)

2. Complete the following derivation of the identify $\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$:

$$\tan(A+B) = \frac{\sin(A+B)}{\cos(A+B)} = \frac{\sin A \cos B + \cos A \sin B}{\cos A \cos B - \sin A \sin B} = \frac{\frac{\sin A \cos B + \cos A \sin B}{\cos A \cos B}}{\frac{\cos A \cos B - \sin A \sin B}{\cos A \cos B}} =$$

3. Compute the following limits (you may *not* use L'Hôpital's rule here):

   (a) $\lim\limits_{x \to 0} \dfrac{\sin(2x)}{x}$ (Hint: $\sin(2x) = 2\sin x \cos x$.)

   (b) $\lim\limits_{x \to \infty} \dfrac{\sin(2x)}{x}$ (Hint: the denominator is going to ∞. What is happening to the numerator?)

   (c) $\lim\limits_{x \to 0} \dfrac{\sin^2 x}{x}$ (Hint: $\sin^2 x = \sin x \cdot \sin x$.)

   (d) $\lim\limits_{x \to 0} \dfrac{x}{\cos x}$

   (e) $\lim\limits_{x \to 0} \dfrac{x}{\sin x}$

   (f) $\lim\limits_{x \to 0} x \cot x$ (Hint: $\cot x = \frac{\cos x}{\sin x}$. Use part (e).)

4. Prove the following identities. In each case, you must start from the expression on the left and arrive at the right. You may not manipulate both sides. Try rewriting each in terms of $\sin x$ and $\cos x$.

   (a) $\csc^2 x - \cot^2 x = 1$

   (b) $\frac{\sin^2 x}{\cos x} + \cos x = \sec x$

   (c) $\cos \theta \tan \theta \csc \theta = 1$