Stuff We Already Know

- Given a right-angled triangle with angle x, write down the definitions of the following:

$$\sin x = \underline{\qquad\qquad\qquad}, \quad \cos x = \underline{\qquad\qquad\qquad}$$

$$\tan x = \underline{\qquad\qquad\qquad}, \quad \sec x = \underline{\qquad\qquad\qquad}$$

$$\csc x = \underline{\qquad\qquad\qquad}, \quad \cot x = \underline{\qquad\qquad\qquad}$$

- Write down the following limits:

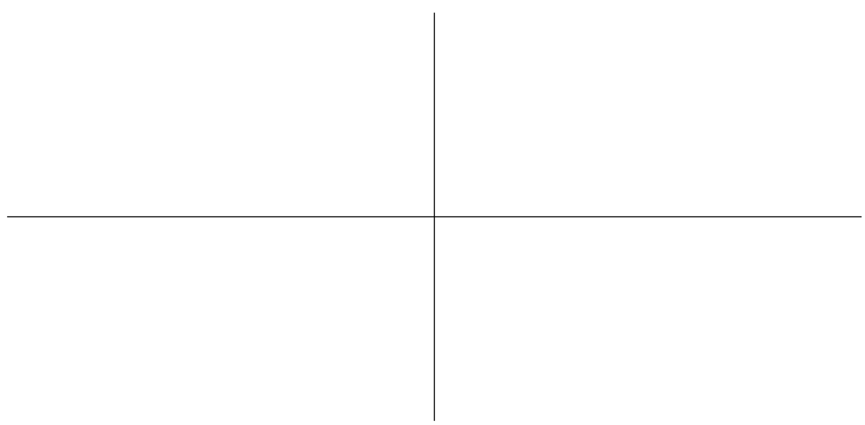
$$\lim_{x\to 0} \frac{\sin x}{x} = \underline{\qquad}\ , \qquad \lim_{x\to 0} \frac{\cos x - 1}{x} = \underline{\qquad}.$$

- Write down the definition of the derivative for a function $f(x)$:

Derivatives of sin and cos

Questions

1. (a) Use Geogebra to graph the function, $g(x) = \frac{\sin(x+0.001)-\sin(x)}{0.001}$ on the axes below with domain $[-2\pi, 2\pi]$. What does this approximate?



   (b) What other function that we know does this graph look like? Graph it on top of your graph of $g(x)$ using Geogebra to verify this.

2. (a) Write down a formula (using a limit) that would give the derivative of $\sin x$:

$$\frac{d}{dx}(\sin x) = \lim \underline{\qquad\qquad\qquad\qquad}.$$

(b) Use the fact that $\sin(A+B) = \sin A \cos B + \cos A \sin B$ to rewrite the derivative of $\sin(x)$ as the sum of two different limits.

(c) Use your two limits to find what $\frac{d}{dx} \sin x$ should be.

3. Use the same sorts of tricks to find $\frac{d}{dx} \cos x$:
(Note: $\cos(A+B) = \cos A \cos B - \sin A \sin B$.)

Other Trig Derivatives

Questions

4. Show that $\frac{d}{dx} \tan x = \sec^2 x$. (Hint: Quotient Rule)

5. Show that $\frac{d}{dx} \sec x = \sec x \tan x$, $\frac{d}{dx} \csc x = -\csc x \cot x$, and $\frac{d}{dx} \cot x = -\csc^2 x$. (Hint: for example, write $\sec x = \frac{1}{\cos x} = (\cos x)^{-1}$ and use the chain rule...)

Questions

6. Find the equation of the tangent line to the graph of $f(x) = \tan x$ at $x = \frac{\pi}{4}$.

7. A particle is moving along a straight line. Its position from its origin (in meters) at time t seconds is given by $s(t) = \sin^2 t$. Find its velocity and acceleration at time $t = 2$ seconds. At what times is the particle at rest? (Hint: The identity $\sin 2x = 2 \sin x \cos x$ may be useful.)

8. (a) Find the linear approximation to the curve $g(x) = \sec x$ at $x = \frac{5\pi}{6}$.

   (b) Use your answer above to estimate $\sec \frac{11\pi}{12}$.

The derivatives we found above are important. While all trig functions have mutiple forms (e.g. we write $\sec^2(x)$ instead of $\frac{1}{1-\sin^2(x)}$), these derivatives have standard forms. For future reference, write these below:

$\frac{d}{dx} \sin x =$ $\qquad$ $\frac{d}{dx} \cos x =$

$\frac{d}{dx} \csc x =$ $\qquad$ $\frac{d}{dx} \sec x =$

$\frac{d}{dx} \tan x =$ $\qquad$ $\frac{d}{dx} \cot x =$

Homework Exercises

1. (a) Use Geogebra to graph the function $y = \sin(2x) - 2\sin(x)$ over the horizontal range $[0, 2\pi]$[1]. Insert special points on your graph to find decimal values of the points where the tangent line to the curve is horizontal.

   (b) Use the derivative to find the exact x-coordinates of all points on the interval $[0, 2\pi]$ where the tangent line to the graph of the function $y = \sin(2x) - 2\sin(x)$ is horizontal. What is the period of this function?

   (You will find the identity $\cos(2x) = 2\cos^2(x) - 1$ useful.)

2. Consider a particular point on a vibrating string as it moves vertically up and down. The position of this point (in mm) at time t (in seconds) is given by

$$s(t) = 10 + \frac{1}{4}\sin(10\pi t).$$

   (a) What is the period of oscillation of this point on the vibrating string?

   (b) Find a formula for the velocity of the point on the string after t seconds.

   (c) Describe the position and the motion (up or down) of this point on the string at $t = 0$ and $t = 0.3$ seconds. (Your answers should have units.)

3. If a projectile is fired from ground level with initial velocity v_0 and inclination angle α and if air resistance can be ignored, the horizontal distance (in feet) it travels is

$$R = \frac{1}{16}v_0^2 \sin(\alpha)\cos(\alpha).$$

   (a) Assuming that a soccer player kicks the ball at a $40°$ angle of inclination, find the initial velocity needed for a kick of 120 feet. (No calculus needed.)

   (b) What value of α maximizes R?

[1] If you don't remember how to use Geogebra, look back to the Derivatives and Roots lab from Math 105L!