

Blast from the Past - The Derivative of $\ln(x)$

Let $y = \ln(x)$.

1. (a) Solve this equation for x .

- (b) Use implicit differentiation to find $\frac{dy}{dx}$ in terms of y .

- (c) Use part (a) and (b) to show that $\frac{d}{dx} \ln(x) = \frac{1}{x}$.

Some Stuff from Last Time...

Last time, we looked at inverse trig functions. Today, we will differentiate them...

2. (a) What are the domain and range of $\arctan x$?

- (b) If $y = \arctan x$, then _____ = x .

Differentiating...

3. (a) Use implicit differentiation on the equation in question 2 to find $\frac{dy}{dx}$ in terms of y .

- (b) Starting from the identity $\sin^2 y + \cos^2 y = 1$, show that $\tan^2 y + 1 = \sec^2 y$.

- (c) Show that $\frac{d}{dx} \arctan x = \frac{1}{1+x^2}$.

4. Use a similar process to the above, this time using the identity $\sin^2 y + \cos^2 y = 1$ directly to find the derivatives of $\sin^{-1} x$ and $\cos^{-1} x$.

Record your results here:

$$\frac{d}{dx} \sin^{-1} x = \underline{\hspace{2cm}} \quad \frac{d}{dx} \cos^{-1} x = \underline{\hspace{2cm}}$$

5. Find the derivatives of the functions below:

(a) $g(y) = \sin(\arcsin y)$

(b) $p(t) = t \sin^{-1} t$

6. At what value(s) of x does the graph of $\sin^{-1}x$ have vertical tangents?
(Note: show this using the derivative of $\sin^{-1}x$, but recall that we also stated the answer last time when we drew the graph of $\sin^{-1}x$. Can you recall how we got there?)
7. Use the derivative of $\tan^{-1}x$ to show that $\tan^{-1}x$ has horizontal asymptotes as $x \rightarrow \infty$ and as $x \rightarrow -\infty$. (Hint: If a function has a horizontal asymptote, what must be true of its derivative as $x \rightarrow \infty$ or $-\infty$?)
8. Use a linear approximation to estimate $\arctan(1.1)$. (Hint: What is $\arctan(1)$?) Is your approximation an overestimate of the true value of $\arctan(1.1)$ or an underestimate of it?