

Why Measure Area Under A Curve?

1. Suppose a car is traveling at a constant velocity of 45 miles/hour.

(a) Draw the graph of $v(t)$, the velocity of the car at time t , where $0 \leq t \leq 10$ hours.

(b) How far does the car travel between $t = 2$ and $t = 7$ hours? Write out your calculation in detail.

(c) What is the area under the graph of $v(t)$ between $t = 2$ and $t = 7$ hours? Again, write out your calculation in detail.

(d) Look at your previous two calculations. What do you notice?

Hypothesis Complete the following hypothesis: the area under the _____ curve between two time points gives the _____ between those two times.

2. Now suppose instead that a particle traveling along a straight line has constant acceleration 2 miles/hr².

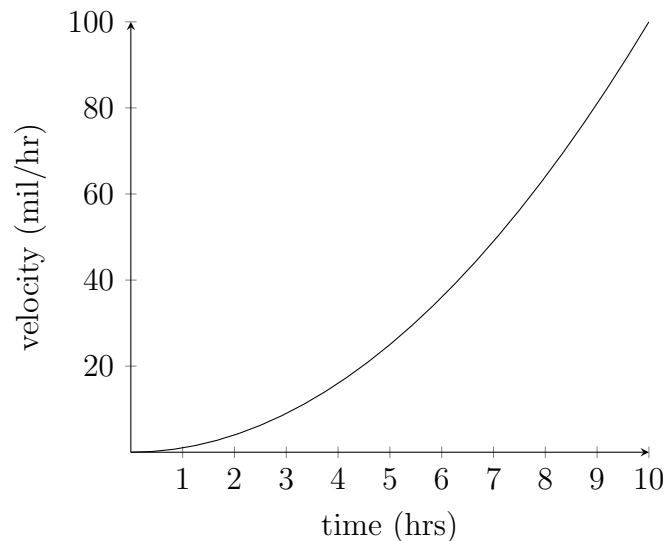
(a) Compute $v(t)$, the velocity of the particle at time t . Assume the car starts from a standstill.

(b) Draw the graph of $v(t)$ for $0 \leq t \leq 10$ hours. Assuming your above hypothesis is correct, shade in an area giving the distance traveled between $t = 2$ and $t = 7$ hours.

(c) Assuming your hypothesis above is correct, how far does the particle travel between $t = 2$ and $t = 7$ hours?

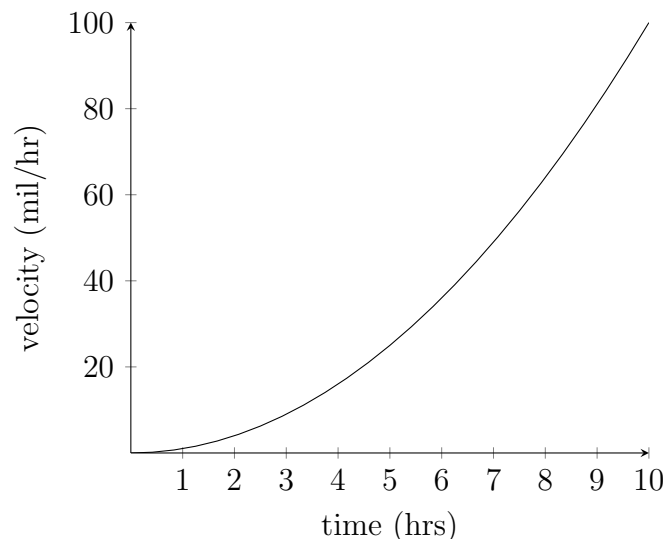
3. Suppose now that $v(t) = t^2$ miles/hour.

(a) On the graph of $v(t)$ for $0 \leq t \leq 10$ hours below, shade in the area giving the distance the particle traveled between $t = 2$ and $t = 7$ hours. Note: we cannot yet calculate this area exactly!

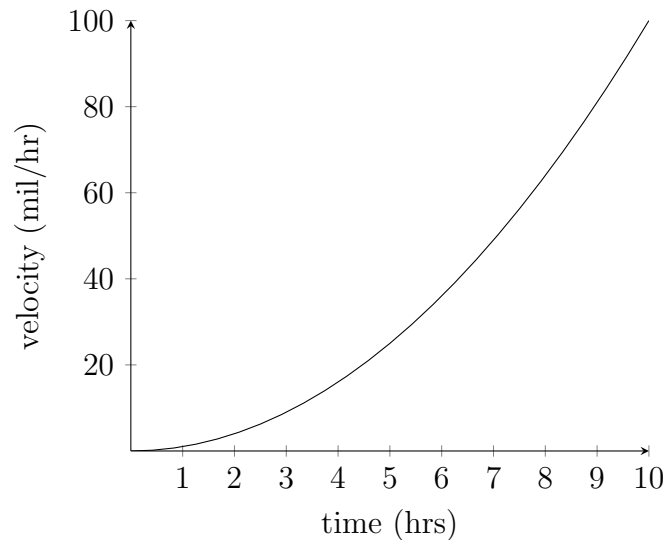


But... ...we can estimate it.

- (b) How fast is the object moving at $t = 2$ hours?
- (c) Suppose for now that the object continues to travel at this velocity for the entire next hour. How far will it have traveled?
- (d) How fast is the object moving at $t = 3$ hours?
- (e) Suppose for now that the object continues to travel at this velocity for the entire next hour. How far will it have traveled over that hour?
- (f) How far does your estimate tell you the particle will have traveled in total since $t = 2$ hours?
- (g) Continue your calculations to estimate how far the object will have traveled between $t = 2$ and $t = 7$ hours.
- (h) On the graph of $v(t)$ below, draw rectangles on your curve whose areas correspond to your hourly estimates above. Is your distance traveled estimate greater than the true distance traveled or less than it? Explain.

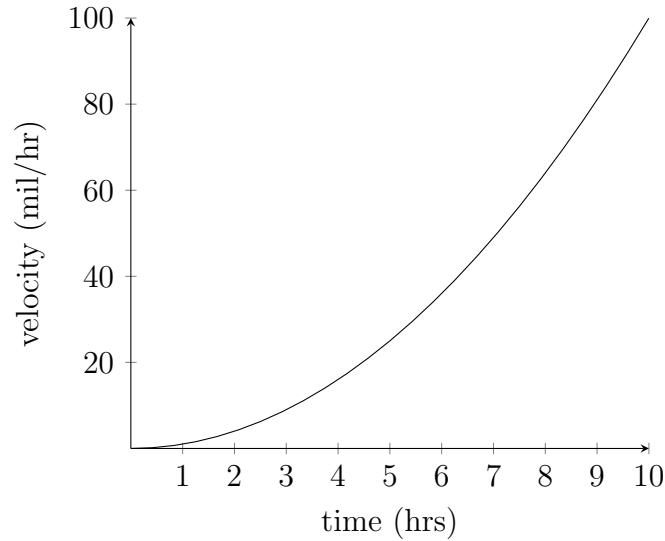


4. Suppose that at time $t = 3$ hours, you realize you forgot to measure how fast you were going at $t = 2$ hours.
- (a) You measure at $t = 3$ hours and decide to estimate that you'd been traveling at that speed for the last hour (i.e. from $t = 2$ to $t = 3$ hours). Estimate how far you traveled.
- (b) You continue to estimate like this (e.g. at $t = 4$ hours, you measure, and use that speed as your estimate for the time period $t = 3$ to $t = 4$ hours). Continue to carry out this calculation to get another estimate of how far you traveled between $t = 2$ and $t = 7$ hours.
- (c) On this next graph of $v(t)$ here, illustrate your latest estimate by drawing appropriate rectangles on your graph and shading them in. Is your new estimate greater or less than the true distance traveled?



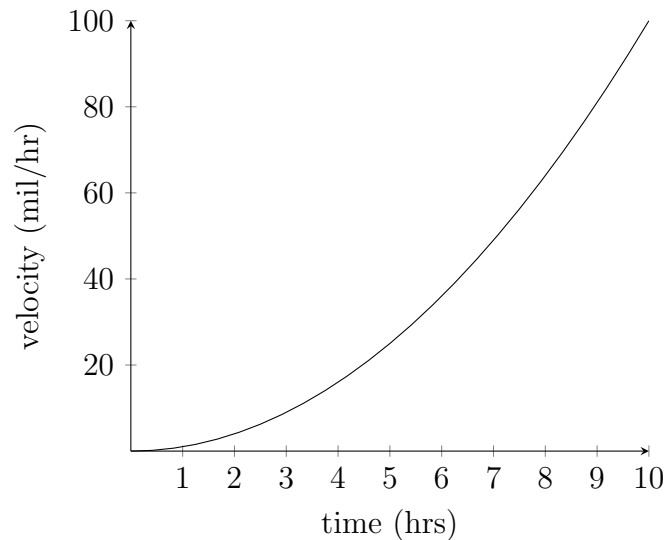
5. Suppose now that you measured your velocity every 30 minutes (0.5 hours) instead of every hour.
- (a) Assuming you're using the first method of estimating above, estimate how far you traveled between $t = 2$ hours and $t = 7$ hours. (Hint: you will need to make 10 calculations and add them up.)

(b) Illustrate your calculation on the graph of $v(t)$:



(c) Is your estimate greater than or less than the true distance traveled? Explain.

(d) Now carry out the same calculation using the second method, and illustrate your estimate. Is it greater than or less than the true distance traveled? Explain.



6. Are your estimates on using 10 intervals page better than the previous ones you made using 5? Explain. How could you make them even better?