Review

1. - If $f(x)$ is *decreasing* on $[a, b]$, then using a left-hand sum to estimate the area 'under' the graph of $f(x)$ between $x = a$ and $x = b$ gives an ____________ of the true area, and using a right-hand sum gives an ____________ of the true area.
   - If $f(x)$ is *increasing* on $[a, b]$, then using a left-hand sum to estimate the area 'under' the graph of $f(x)$ between $x = a$ and $x = b$ gives an ____________ of the true area, and using a right-hand sum gives an ____________ of the true area.
   - Draw examples to show why the above are true.

2. - To estimate the area, we divide $[a, b]$ into n equal subintervals of width $\Delta x =$ ______.
   - In a left-hand sum, the area of the i^{th} rectangle is ____________.
   - In a right-hand sum, the area of the i^{th} rectangle is ____________.

3. Consider the function $f(x) = (x - 1)(x - 2)$. Draw two copies of a graph of this function, then draw a left-hand sum and a right-hand sum with $n = 3$ subintervals for the area 'under' the graph between $x = 0$ and $x = 1.5$. Compute both sums.

4. Why did I put 'under' in quotes?

The Midpoint and Trapezoid Sums

Definition In a *midpoint sum*, we choose to compute the height of each rectangle in the *center* of each subinterval instead of on the left or right.

5. Fill in the blanks:
   - When we divide the interval $[a, b]$ into n equal subintervals, the left hand of the i^{th} subinterval is ______, and its right hand is ______.
   - So the center of the i^{th} subinterval is ______.
   - The area of the i^{th} rectangle in a midpoint sum is therefore ____________.

Definition The *trapezoid sum* is the average of the left hand and right hand sums.

6. Draw and then calculate $(LHS)_2$, $(RHS)_2$, $(TRAP)_2$ and $(MPS)_2$ for the following areas:

   (a) The area 'under' $x^2 - 1$ between $x = 0$ and $x = 4$.

   $(LHS)_2 =$ ______ $(RHS)_2 =$ ______ $(TRAP)_2 =$ ______ $(MPS)_2 =$ ______

   (b) The area 'under' $\sqrt{x} - 2$ between $x = 0$ and $x = 4$.

   $(LHS)_2 \approx$ ______ $(RHS)_2 \approx$ ______ $(TRAP)_2 \approx$ ______ $(MPS)_2 \approx$ ______

(c) Assuming one of $TRAP_2$ and MPS_2 is an overestimate of the true area, and the other an underestimate, which one is which in each of the two cases above?

(d) When does it seem the MPS and the average of the LHS and RHS are over/underestimates of the true area?

7. Let's demonstrate why your answer to the last question is true. To do this, we must recast these two sums as sum of areas of *trapezoids*:

Conclusion

8.
- If $f(x)$ is ____________, then LHS_n is an underestimate of the true 'area' under $f(x)$ and RHS_n is an overestimate for it.
- If $f(x)$ is ____________, then LHS_n is an overestimate of the true 'area' under $f(x)$ and RHS_n is an underestimate for it.
- If $f(x)$ is ____________, then MPS_n is an underestimate of the true 'area' under $f(x)$ and $TRAP_n = \frac{LHS_2+RHS_2}{2}$ is an overestimate for it.
- If $f(x)$ is ____________, then MPS_n is an overestimate of the true 'area' under $f(x)$ and $TRAP_n = \frac{LHS_2+RHS_2}{2}$ is an underestimate for it.