Throughout this worksheet, assume $f(t)$ and $g(t)$ are continuous.

1. $\int_a^a f(t)\,dt = 0.$

2. Let c be a constant, then $\int_a^b cf(t)\,dt = c\int_a^b f(t)\,dt.$

3. $\int_b^a f(t)\,dt = -\int_a^b f(t)\,dt$

4. $\displaystyle\int_a^b f(t) + g(t)\ dt = \int_a^b f(t)\ dt + \int_a^b g(t)\ dt.$

5. $\displaystyle\int_a^b f(t) - g(t)\ dt = \int_a^b f(t)\ dt - \int_a^b g(t)\ dt.$

6. $\int_a^b f(t)\ dt = \int_a^c f(t)\ dt + \int_c^b f(t)\ dt.$

7. If $m \leq f(t) \leq M$ for all values of t in $[a, b]$ then $m(b-a) \leq \int_a^b f(t)\,dt \leq M(b-a)$.

8. If $f(t) \leq g(t)$ for all values of t in $[a, b]$ then $\int_a^b f(t)\,dt \leq \int_a^b g(t)\,dt$

Another Riemann Sums Exercise

Consider the Riemann sum $\sum_{k=0}^{499} \left(5-(2+0.006k)^2\right)\Delta x$.

9. What are the values of Δx and a? (Hint: somewhere in that expression is $a + k\Delta x$. Find it to get values for a and Δx.)

10. What definite integral is being approximated? (Hint: you will need to find b, and $f(x)$.)

11. Is this Riemann sum an overestimate of the integral, an underestimate of it, or is it impossible to tell? Explain your answer.

12. Suppose we write another Riemann sum to approximate the same definite integral. This new Riemann sum has the form

$$\sum_{k=0}^{9999} \left(5-(2+\underline{\hspace{3cm}})^2\right)\Delta x.$$

   (a) What is the new value of Δx?

   (b) Fill in the blank in the new Riemann sum.

   (c) Is this new Riemann sum an overestimate or an underestimate of the definite integral? Explain.

   (d) Is this new Riemann sum smaller or larger than the first one? Explain.