

Review

1. (a) The Chain Rule states that:

$$\frac{d}{dx} [f(g(x))] =$$

- (b) By taking the antiderivatives of both sides, we get:

$$\int f'(g(x))g'(x) dx =$$

Method of u -substitution

2. (a) Suppose we want to evaluate $\int f'(g(x))g'(x) dx$. Let $u = g(x)$.

Then $du = \underline{\hspace{2cm}} dx$.

- (b) Hence,

$$\begin{aligned} \int f'(g(x))g'(x) dx &= \\ &= \\ &= \end{aligned}$$

- (c) Note also that

$$\begin{aligned} \int_a^b f'(g(x))g'(x) dx &= \\ &= \end{aligned}$$

Examples

Note: Not all of these require substitutions. One of the most important integration skills is spotting what is the easiest method to use for a given integral.

3. $\int x(1+x^2)^5 dx$

$$4. \int_0^{\frac{\pi}{2}} \sin x \cos x \, dx$$

$$5. \int_0^1 \frac{x}{\sqrt{x^2+1}} \, dx$$

$$6. \int \frac{1}{3x+1} \, dx$$

$$7. \int \frac{1}{e^{3x}} \, dx$$

8. $\int \sin^2 x \, dx$ (Hint: $\cos 2x = 1 - 2 \sin^2 x$)

9. $\int_0^2 \frac{e^x}{1 + e^{2x}} \, dx$ (Hint: What's another way to write e^{2x} ? Rules of exponentials...)

10. $\int \frac{\ln x}{x} \, dx$

11. $\int \cos^2 x \sin x \, dx$

12. $\int x e^{-x^2} dx$

13. $\int \tan x dx$ (Hint: write $\tan x$ as a quotient of two other trig functions.)

14. $\int \frac{2x}{\sqrt{1-x^4}} dx$

15. $\int_0^1 x\sqrt{x+1} dx$ (Hint: Let $u = x + 1$. Then $x = \dots$?)