The Product Rule:

1. (a) The Product Rule states that:

$$\frac{d}{dx}\left[u(x)v(x)\right] = \underline{\hspace{6cm}}$$

   (b) Thus, we can apply the Fundamental Theorem of Calculus and obtain

$$u(x)v(x) = \int \qquad\qquad\qquad + \int \qquad\qquad\qquad .$$

   (c) We can now write $du = u'(x)\ dx$ and $dv = v'(x)\ dx$. Rearranging the terms, we get the

Integration by Parts Formula:

In order for this to be useful, the integral on the right needs to be easier than the integral on the left. How do we pick u and dv? Well, to start with, dv should to be easy to integrate!

Examples:

2. $\displaystyle\int xe^x\ dx$

   $u =$ $dv =$
   $du =$ $v =$

3. $\displaystyle\int_1^2 \ln x\ dx$

   $u =$ $dv =$
   $du =$ $v =$

4. $\displaystyle\int x\ln x\ dx$

5. $\displaystyle\int_0^{\pi} x \sin x \, dx$

6. $\displaystyle\int_0^{1} x\sqrt{x+1} \, dx$ (Compare to worksheet 7-2, Q15. Which method do you prefer?)

7. $\displaystyle\int_1^{2} x^2 e^x \, dx$ (Hint: Integrate by parts twice)

8. $\displaystyle\int \arctan x \, dx$ (Hint: Let $u = \arctan x$.)

9. $\int e^x \cos x \, dx$ (Hint: Integrate by parts twice, and carefully observe the equation you obtain.)

10. $\int x(\ln x)^3 \, dx$ (Hint: Integrate by parts times.)

11. $\int xe^{-x^2} \, dx$ (Hint: Don't work too hard!)