Force and Work

Purpose:

The purpose of this lab is to understand the physical concept of *work*, a measure of the amount of energy transferred in or out of a system as it is moved by a force. We will start from a simple example that does not require any calculus, and proceed to develop a Riemann sum that calculates work more generally.

The concept of work[1]

> "The mass contained in a closed system is a conserved quantity, but if the system is not closed, we also have ways of measuring the amount of mass that goes in or out. The water company does this with a meter that records your water use.

> Likewise, we often have a system that is not closed, and would like to know how much energy comes in or out. Energy, however, is not a physical substance like water, so energy transfer cannot be measured with the same kind of meter. How can we tell, for instance, how much useful energy a tractor can "put out" on one tank of gas?

> The law of conservation of energy guarantees that all the chemical energy in the gasoline will reappear in some form, but not necessarily in a form that is useful for doing farm work. Tractors, like cars, are extremely inefficient, and typically 90% of the energy they consume is converted directly into heat, which is carried away by the exhaust and the air flowing over the radiator. We wish to distinguish the energy that comes out directly as heat from the energy that serves to accelerate a trailer or to plow a field, so we define a technical meaning of the ordinary word "work" to express the distinction.

> Definition of work: Work is the amount of energy transferred into or out of a system, not counting energy transferred by heat conduction."
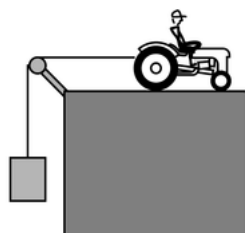

Figure 1: The tractor raises the weight over the pulley, increasing its gravitational potential energy.

Figure 2: The tractor accelerates the trailer, increasing its kinetic energy.

Note that in both these examples, a force is involved, and the object the force is applied to is moved through some distance. If we increase the distance, we do **more/less** work (cross out the wrong answer). If we apply a greater force, we do **more/less** work (cross out the wrong answer).

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[1]Benjamin Crowell - *Light and Matter*, http://www.lightandmatter.com/html_books/lm/ch13/ch13.html

106L Labs: Force and Work

1. Keeping this in mind, which of the following definitions of work would make sense? (W =work, F =force, d =distance)

$$\text{(a) } W = F/d \quad \text{(b) } W = Fd \quad \text{(c) } W = d/F$$

Definition: The ***work*** done by a constant force F acting through a straight-line distance d is
$$W = Fd.$$

Note that if the force is *not constant*, this formula is *not valid!*

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2. (a) Recall that Newtons' second law of motion says that $F =$ ___ ___, where F is force (measured in Newtons), ___ is _____, measured in _____ and ___ is _____, measured in _____.

   (b) Write one Newton in terms of the units from part (a):

   $$1 \text{ N} = 1\text{\_\_\_ \_\_\_ per \_\_\_}^2.$$

   (c) If force is measured in Newtons (N) and distance in meters (m), what units would work be measured in?

3. If I use 7N of force to pull an object 3 meters, how much work will I have done?

A bit of a discussion on the matter of kilograms and Newtons: A kilogram is a unit of _____, whereas a Newton is a unit of _____. The force exerted by the Earth on a mass of one kilogram is _____ (Hint: use Newton's second law, see question 2(a)). This defines another unit of the force, often referred to as the kilogram-force:

$$1 \text{ kilogram-force } = \text{\_\_\_ N}.$$

You may sometimes see a reference to, say, '2kg of force'. In fact, when I was writing this lab, I made precisely that mistake. It is inaccurate, as the units don't match.

4. If I pick up a weight with mass 1kg and move it up one meter, how much work will I have done, in Newtons? (See paragraph above for a hint!)

5. Draw a graph showing the force applied to the object in Question 3 above at distance x from its starting position. On your graph, illustrate the total work done on the object.

Work done by a non-constant force

Suppose now that we are pulling an object with increasing force. Say that at distance x meters from its starting point, we apply x^2 Newtons to the object. Suppose we again pull the object 3m.

6. By analogy to Question 5 above, conjecture how you might calculate the total work done.

Let's prove the conjecture: Suppose now that the force applied to an object at distance x from some reference point is $F(x)$. Suppose that the object is moved from distance a from the reference point to distance b from the reference point. Divide the distance from a to b into n subintervals, and define $\Delta x = $ _____.

Then the work done in moving the object from distance $a + (k-1)\Delta x$ to distance $a + k\Delta x$ is approximately:
$$W_k = \underline{\hspace{3cm}}.$$

7. Write the approximate total work done as a Riemann sum.

8. By taking the limit as n goes to infinity, write down an integral that expresses the exact total work done by the force. Be sure to integrate with respect to the right variable!

Conclusion:

The **work** done by a force of $F(x)$ at distance x from some reference point in moving an object from $x = a$ to $x = b$ is

$$W = \qquad\qquad .$$

This is the correct formula for work done by a *non-constant* force.

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9. Calculate the work done in moving the above object from $x = 0$ meters to $x = 3$ meters.

Examples

Springs

Recall Hooke's law for linear springs:

Hooke's law:

The force required to stretch a spring from its rest position to a position x is proportional to x. That is:
$$F(x) = \underline{\hspace{0.5cm}}x,$$
where ____ is called the *spring constant*. It measures the *stiffness* of the spring.

Questions:

10. If F is measured in Newtons and x in meters, what are the units of the constant k?

11. Suppose a spring obeys Hooke's law with spring constant $k = 0.1$ ____. Find the work done in:

    (a) Stretching the spring from its rest position to 1 meter.

    (b) Stretching the spring from an extension of 1.5 meters to 2 meters.

12. Suppose a force of 40N is required to keep a spring extended to 4cm.

    (a) Find the spring constant k.

    (b) Find the work required to extend the spring from 4cm to 8cm.

13. Suppose that a mass of 1.5kg is hung from a spring. You find that this mass stretches the spring 10cm. How much work would have to be done to pull the spring until it is extended by a further 15cm?

Flying out of the solar system

How much energy is required for a spaceship to fly out of the solar system? This section will attempt to answer that question.

Newton's law of gravitation:

Suppose we have two bodies of masses m_1 and m_2. Suppose also that the distance between them is r. Then the attractive (gravitational) force between the two object is given by

$$F(r) = -\frac{Gm_1m_2}{r^2},$$

where $G = 6.674 \times 10^{-11}$ N m^2/kg^2 is the universal gravitational constant.

Questions:

14. The sun has mass 2×10^{30} kg. Consider an object of mass of 1000 kg at a distance of 1,500,000 (1.5×10^6) km from the sun.

    (a) Calculate the gravitational force the two bodies exert on one another.

    (b) Use part (a) and Newton's second law of motion to calculate the acceleration of the object and of the sun.

    (c) The two objects exert the same force on each other, namely the force you calculated in part (a). Using part (b), describe the motion of the object and of the sun. Explain in your own words: does this tally with what you think should happen in this situation?

15. Now suppose that object, of mass 1000 kg, is a spaceship, and that it starts above Earth, 150 million km from the sun. Calculate how much work is done in flying the spaceship to each of the following points. You may disregard the gravitational force of every body in the solar system except the sun (e.g. the Earth, Saturn, etc).

    (a) The planet Jupiter, 741 million km from the sun.

    (b) The non-planet Pluto, 3660 million km from the sun.

    (c) The star Alpha Centauri, 43×10^{12} km from the sun.

    (d) Infinitely far away.

Report

Answer the following questions using complete sentences and hand in one report per group.

1. In lab, we wrote that 1 kilogram-force is 9.8 Newtons. That number (9.8) should look familiar.

   (a) Where have you seen it before? What did it mean in that context? What are its units?

   (b) Write down Newton's second law of motion, relating force to mass and acceleration.

   (c) One Newton is defined to be the amount of force required to accelerate a mass of one kilogram at a rate of one meter per second squared. Given this, and your answers to (a) and (b), what units do we arrive at for force? Complete the following:

   $$1 \text{ Newton} = 1\underline{\hspace{3cm}}.$$

   (d) Suppose an apple has mass 102g ($\approx 1/9.8$ kg). Given your answers to (a) and (b), what force (in Newtons) does the Earth's gravitational field exert on the apple?

   (e) Why are we talking about apples, anyway? (Hint: Newton!)

2. One Newton meter is sometimes referred to as one *Joule*[2] i.e.

   $$1Nm = 1J.$$

   (a) The *efficiency* of given fuel (e.g. hydrogen, diesel, etc) is measured in megajoules (MJ) per kilogram. Fill in the following table using data available online:

Fuel	Regular gasoline	Ethanol	Biodiesel	Hydrogen
Efficiency (MJ/kg)				

   (b) Suppose that I want to use regular gasoline to move a car 100 meters. Suppose that I keep pressing the pedal, so that the force applied at distance x meters from the start is $F(x) = 1.31x^2$.

      i. Calculate the total work done (in megajoules) in moving the car 100 meters.

      ii. Assuming the fuel efficiency of gasoline is what you found in the table above, calculate the mass (in grams) of gas you need.

      iii. Find the density of gasoline online and use it to find the volume (in liters) of gas you need to move the car 100 meters.[3]

3. Calculate the amount of work done in each of the following situations, expressing your answers in Newton meters:

   (a) Pulling am object of mass 1000kg up a 700m vertical mineshaft using a cable, whose density is 1.5kg/m.

   (Hint: draw this situation! Note that the mass of the object is constant, but the mass of the cable depends on how much of it is left to pull. You'll need to find a formula for the total mass remaining after you've pulled x meters.)

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[2]Though there are some problems with this: Newton meters are most often used to measure torque, whereas Joules are used to measure energy (=work). See http://en.wikipedia.org/wiki/Newton_metre for details.

[3]There are serious issues with this entire calculation. If you're really interested, see http://en.wikipedia.org/wiki/Fuel_efficiency for details and take some mechanical engineering classes...

      (b) A force of 1200N compresses a spring from its natural length of 18cm to a length of 16cm. How much work is done in compressing it from 16cm to 14cm?

4. You push a 10 ton weight really hard, but it doesn't move.

      (a) What is the total amount of work done on the weight?

      (b) You keep pushing for ages, but the weight doesn't move. However, you feel tired, so some work must have been done. Explain the discrepancy between this conclusion and your result in part (a).